2.12 find $\vec{\nabla} f$ & $\nabla^2 f$ in ~~sph~~ cartesian & spherical:

a) $f = x^2 + y^2 + z^2 = r^2$

cartesian $\quad \vec{\nabla} f = 2x\hat{i} + 2y\hat{j} + 2z\hat{k} = 2\bar{x}$

$\nabla^2 f = 2 + 2 + 2 = 6$

spherical $\quad \vec{\nabla} f = \hat{r}\, \partial f/\partial r + \hat{\theta} \frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\, \partial f/\partial \phi$

$f = r^2 \rightarrow \vec{\nabla} f = \hat{r}\cdot 2r = 2\bar{r}$

$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \partial f/\partial r\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\, \partial^2 f/\partial \phi^2$

$= 1/r^2 \frac{\partial}{\partial r}(2r^3) = 6r^2/r^2 = 6$

b) $f = (x^2 + y^2 + z^2)^{1/2} = r$

$\vec{\nabla} f = 1/2\,(x^2+y^2+z^2)^{-1/2}(2x\hat{i} + 2y\hat{j} + 2z\hat{k}) = \bar{x}/r$

cartesian $\quad \nabla^2 f = \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$= \frac{1}{\sqrt{x^2+y^2+z^2}} + \frac{x\cdot -1/2 \cdot 2x}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{\sqrt{x^2+y^2+z^2}} + \frac{y\cdot -1/2\cdot 2y}{(x^2+y^2+z^2)^{3/2}}$

$+ \frac{1}{\sqrt{x^2+y^2+z^2}} + \frac{z\cdot -1/2\cdot 2z}{(x^2+y^2+z^2)^{3/2}} = \frac{3 - (x^2+y^2+z^2)/(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}}$

$= 2/\sqrt{x^2+y^2+z^2}$

spherical $\quad \vec{\nabla} f = \hat{r}\cdot 1 = \bar{r}/r \quad ; \quad \nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, df/dr\right) + \varnothing = 2/r$

c) $f = (x^2+y^2+z^2)^{-1/2} = 1/r$

cartesian $\quad \vec{\nabla} f = (x^2+y^2+z^2)^{-3/2}\left(\hat{i}\cdot -1/2\cdot 2x + \hat{j}\cdot -\frac{1}{2}\cdot 2y + \hat{k}\cdot -\frac{1}{2}\cdot 2z\right) = -\bar{x}/r^3$

$\nabla^2 f = (x^2+y^2+z^2)^{-3/2}\left[-3 + \frac{-1}{x^2+y^2+z^2}\left(-\frac{3}{2}\right)(2x^2+2y^2+2z^2)\right] = \varnothing$

$\vec{\nabla} f = \hat{r}\, \partial f/\partial r + \varnothing = \hat{r}\,(-1/r^2) = \vec{r}/r^3$

$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \partial f/\partial r\right) + \varnothing = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot -\frac{1}{r^2}\right) = \frac{-1}{r^2}\frac{\partial}{\partial r}(1) = \varnothing$