2.13 Calculate divergence & curl of:

a) $\vec{x}$ $\quad \bar{\nabla}\cdot\bar{x} = \partial x/\partial x + \partial y/\partial y + \partial z/\partial z = 3$

cartesian $\quad \bar{\nabla}\times\bar{x} = \hat{i}\left(\partial y/\partial z - \partial z/\partial y\right) + \hat{j}\left(\partial z/\partial x - \partial x/\partial z\right) + \hat{k}\left(\frac{\partial x}{\partial y} - \partial y/\partial x\right) = \emptyset$

$\bar{\nabla}\cdot\bar{x} = \cancel{\partial/\partial x\ \partial D/\partial r} \quad 1/r^2\ \partial/\partial r\left(r^2\, r\right) = \frac{1}{r^2}\cdot 3r^2 = 3$

spherical $\quad \bar{\nabla}\times\bar{x} = \frac{1}{r\sin\theta}\,\partial r/\partial\phi\ \hat{\theta} - \frac{1}{r}\,\partial r/\partial\theta\ \hat{\phi} = \emptyset$

b) $\bar{x}/r$: $\quad \bar{\nabla}\cdot\bar{x} = \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$= \frac{3}{\sqrt{x^2+y^2+z^2}} + \frac{x\cdot -1/2\cdot 2x + y\cdot -1/2\cdot 2y + z\cdot -\frac{1}{2}\cdot 2z}{\left(x^2+y^2+z^2\right)^{-3/2}}$

$= 2/\sqrt{x^2+y^2+z^2}$

$\bar{\nabla}\times\bar{x}/r = \hat{i}\left[\partial/\partial y\left(z/r\right) - \frac{\partial}{\partial z}\left(y/r\right)\right] + \hat{j}\left[\frac{\partial}{\partial z}\frac{x}{r} - \frac{\partial}{\partial x}\frac{z}{r}\right] + \hat{k}\left[\frac{\partial}{\partial x}y/r - \frac{\partial}{\partial y}\frac{x}{r}\right]$

$= \hat{i}\left[\frac{z}{r^3}\cdot\frac{-1}{2}\cdot 2y - \frac{y}{r^3}\cdot\frac{-1}{2}\cdot 2z\right] + \dots = \emptyset$

c) $\hat{k}\times\bar{x} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ x & y & z \end{vmatrix} = -\hat{i}y + \hat{j}x$

$\bar{\nabla}\cdot(\hat{k}\cdot\bar{x}) = -\partial y/\partial x + \partial x/\partial y = \emptyset$

$\bar{\nabla}\times(\hat{k}\cdot\bar{x}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & x & 0 \end{vmatrix} = 0 + 0 + (1 - -1)\hat{k} = 2\hat{k}$

$\hat{k}\times\bar{r} = r\sin\theta\ \hat{\phi}$

$\bar{\nabla}\cdot(\hat{k}\times\bar{r}) = \frac{1}{r\sin\theta}\frac{\partial F}{\partial\phi} = \emptyset$

$\bar{\nabla}\times(\hat{k}\times\bar{r}) = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\ r\sin\theta\right)\hat{r} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot r\sin\theta\right)\hat{\theta}$

$= 2\cos\theta\ \hat{r} - 2\sin\theta\ \hat{\theta}\ .\ (= 2\hat{k})$