2.17 $\bar{H}(\bar{x}) = x^2 y\,\hat{i} + y^2 z\,\hat{j} + z^2 x\,\hat{k}$.

find $\vec{F}$, $\bar{G}$ if $\bar{H} = \bar{F} + \bar{G}$

$\bar{\nabla}\times\bar{G} = \bar{\nabla}\times\bar{H} = -y^2\,\hat{i} - z^2\,\hat{j} - x^2\,\hat{k}$

Try $\bar{\nabla}\times\bar{A} = \bar{G}$, $\bar{\nabla}\cdot\bar{A} = \emptyset \rightarrow -\nabla^2\bar{A} = \emptyset$

$\bar{A} = y^4/12\,\hat{i} + z^4/12\,\hat{j} + x^4/12\,\hat{k}$

$\bar{G} = \bar{\nabla}\times\bar{A} = -z^3/3\,\hat{i} - x^3/3\,\hat{j} - y^3/3\,\hat{k}$

$\bar{\nabla}\cdot\bar{F} = \bar{\nabla}\cdot\bar{H} = 2xy + 2yz + 2zx$

Try $\vec{F} = -\bar{\nabla}\psi$ or $\psi = \frac{-x^3 y}{3} - \frac{y^3 z}{3} - \frac{z^3 x}{3}$

$\bar{F} = -\bar{\nabla}\psi = (x^2 y + z^3/3)\hat{i} + (y^2 z + \frac{x^3}{3})\hat{j} + (z^2 x + \frac{y^3}{3})\hat{k}$

2.18 $\bar{H} = -\bar{\nabla}\psi + \bar{\nabla}\times\bar{A}$. show $\nabla^2\psi = -\bar{\nabla}\cdot H$

$\bar{\nabla}(\bar{\nabla}\cdot\bar{A}) - \nabla^2\bar{A} = \bar{\nabla}\times\bar{H}$

$\bar{\nabla}\cdot\bar{H} = \bar{\nabla}\cdot(-\bar{\nabla}\psi + \bar{\nabla}\times\bar{A}) = \bar{\nabla}\cdot(-\bar{\nabla}\psi) = -\nabla^2\psi$

$\bar{\nabla}\times\bar{H} = \bar{\nabla}\times(\bar{\nabla}\times\bar{A}) = \bar{\nabla}(\bar{\nabla}\cdot\bar{A}) - \bar{\nabla}^2\bar{A}$

2.19 $\bar{F}(\bar{x}) = C\,\bar{x}$

a) $\bar{\nabla}\cdot\bar{F} = \frac{\partial}{\partial x}(Cx) + \frac{\partial}{\partial y}(Cy) + \frac{\partial}{\partial z}(Cz) = 3C$

b) $\oint \bar{F}\cdot d\bar{A} = \int_V \bar{\nabla}\cdot\bar{F}\, d^3x = 3C\int_{-\ell/2}^{\ell/2} dx \int_{-\ell/2}^{\ell/2} dy \int_{-\ell/2}^{\ell/2} dz$

$= 3C\,\ell^3$