

$$2.20 \quad \vec{G} = \hat{k} \times \vec{x} \quad a)$$

$$= -y \hat{i} + x \hat{j} = r \hat{\phi}$$



$$b) \text{ find } \oint \vec{G} \cdot d\vec{l} = \oint (\nabla \times \vec{G}) \cdot d\vec{A}$$

$$\nabla \times \vec{G} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & x & 0 \end{vmatrix} = -\hat{j} + z \hat{k}$$

$$d\vec{A} = dA \hat{k} \rightarrow \oint (\nabla \times \vec{G}) \cdot d\vec{A} = z \int dA = 2\pi r^2$$

if $r=1$, $= 2\pi$

$$2.21 \quad \vec{F}(\vec{x}) = f(r) \vec{x}, \quad r = |\vec{x}|$$

$$a) \nabla \times \vec{F} = f \underbrace{\nabla \times \vec{x}}_{=0} + \nabla f \times \vec{x}$$

$\nabla f = \frac{df}{dr} \hat{r}, \quad \nabla f \times \vec{x} = \frac{df}{dr} \hat{r} \times r \hat{r} = 0$

$$b) \nabla \cdot \vec{F} = 0 \text{ also.}$$

$$\nabla \cdot \vec{F} = \nabla f \cdot \vec{x} + f (\nabla \cdot \vec{x}) = \hat{r} \frac{df}{dr} \cdot \vec{x} + 3f = r \frac{df}{dr} + 3f$$

$$\text{or } df/dr = -3f/r$$

$$f = kr^{-3}$$