2-23 Prove: a) $\bar{A}\cdot\bar{\nabla}(1/r) = -\bar{A}\cdot\bar{x}/r^3$

$$\bar{A}\cdot\bar{\nabla}(1/r) = \bar{A}\cdot(-1/r^2)\hat{r} = -\vec{A}\cdot\vec{r}/r^3$$
$$= -\vec{A}\cdot\vec{x}/r^3$$

b) $\bar{B}\cdot\bar{\nabla}\left[\bar{A}\cdot\bar{\nabla}(1/r)\right] = \dfrac{3(\bar{A}\cdot\bar{x})(\bar{B}\cdot\bar{x})}{r^5} - \dfrac{\bar{A}\cdot\bar{B}}{r^3}$

$$\bar{\nabla}\left[\bar{A}\cdot\bar{\nabla}(1/r)\right] = \bar{\nabla}\left(-\frac{\bar{A}\cdot\bar{x}}{r^3}\right)$$
$$= -\bar{A}\cdot\bar{x}\,\bar{\nabla}(1/r^3) - \frac{\bar{\nabla}(\bar{A}\cdot\bar{x})}{r^3}$$
$$= \frac{+3\bar{A}\cdot\bar{x}\,\hat{r}}{r^3} - \frac{\bar{A}}{r^3}$$

$$\bar{B}\cdot\bar{\nabla}\left[\bar{A}\cdot\bar{\nabla}(1/r)\right] = \bar{B}\cdot\left[\frac{-\bar{A}}{r^3} + \frac{3\bar{A}\cdot\bar{x}\,\hat{r}}{r^3}\right]$$
$$= \frac{3(\bar{A}\cdot\bar{x})(\bar{B}\cdot\bar{x})}{r^5} - \frac{\bar{A}\cdot\bar{B}}{r^3}$$

2-24 $\vec{F}(\bar{x}) = \hat{\varphi}$.

a) $\oint_c \bar{F}\cdot d\ell$, c is the circle of radius about the origin

$$\oint_c \hat{\varphi}\cdot r\,d\varphi\,\hat{\varphi} = r\int_0^{2\pi} d\varphi = 2\pi r$$

b) $\int_H \bar{\nabla}\times\bar{F}\cdot d\bar{A}$, H the upper spherical hemisphere.

$d\bar{A} = r^2\sin\theta\,d\theta\,d\varphi\,\hat{r}$, $(\bar{\nabla}\times\bar{F})\cdot d\bar{A} = (\bar{\nabla}\times\bar{F})_r\,dA_r$

$$(\bar{\nabla}\times\bar{F})_r = \frac{1}{r\sin\theta}\left[\partial/\partial\theta\,(\sin\theta)\right] = \frac{\cos\theta}{r\sin\theta} = \frac{\cot\theta}{r}$$

$$\int_H \bar{\nabla}\times\bar{F}\cdot d\bar{A} = \int\frac{\cot\theta}{r}\cdot r^2\sin\theta\,d\theta\,d\varphi = r\underbrace{\int_0^{\pi/2}\cos\theta\,d\theta}_{1}\int_0^{2\pi}d\varphi = 2\pi r$$

c) Over a disk, $d\bar{A} = \rho\,d\rho\,d\varphi\,\hat{k}$ in cylindrical system

$$(\bar{\nabla}\times\bar{F})\cdot\hat{k} = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho)\right] = 1/\rho$$

$$\int_D \bar{\nabla}\times\bar{F}\cdot d\bar{A} = \int\frac{1}{\rho}\,\rho\,d\rho\,d\varphi = \int_0^r d\rho\int_0^{2\pi}d\varphi = 2\pi r$$