2-26 $\Phi(\bar{x}) = \bar{C}\cdot\bar{x}/r^3$, $\bar{C} = C\hat{k}$

$$\phi = \frac{Cz}{(x^2+y^2+z^2)^{3/2}} = \frac{C\cos\theta}{r^2}$$

a) Find $\vec{E}(\bar{x}) = -\bar{\nabla}\phi$

In spherical coordinates

$$-\bar{\nabla}\phi = -\left(\hat{r}\,\partial\phi/\partial r + \hat{\theta}\,\frac{1}{r}\,\partial\phi/\partial\theta\right) \quad \text{since } \frac{\partial\phi}{\partial\varphi} = 0$$

$$= -\hat{r}\,\frac{C\cos\theta}{r^3}\cdot -2 \;-\; \hat{\theta}\,\frac{1}{r}\,\frac{C}{r^2}\sin\theta\cdot -1$$

$$= \frac{C}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

b) On the x-axis, $\theta = \pi/2, \varphi = 0$ & $\cos\theta = 0$

$$\hat{E} = \hat{\theta} = -\hat{k}$$

c) On the z-axis, $\theta = 0$,

$$\hat{E} = \hat{r} = +\hat{k}$$

d) 

2.27 $\bar{F}(\bar{x}) = \bar{x}/r^3$

a) $\Phi = \oint \bar{F}\cdot d\bar{A} = \int \frac{a\hat{r}}{a^3}\cdot\hat{r}\,a^2\sin\theta\,d\theta\,d\phi = \frac{4\pi a^2}{a^2} = 4\pi$

b) Through circular disk at height $H = z$

$$\Phi = \oint_{\text{Disk}} \bar{F}\cdot d\bar{A} = \oint \frac{\rho\hat{\rho} + H\hat{k}}{(\rho^2+H^2)^{3/2}}\cdot\hat{k}\,\rho\,d\rho\,d\phi$$

$$= \oint \frac{H}{(\rho^2+H^2)^{3/2}}\,\rho\,d\rho\,d\phi = 2\pi H\int_0^a \frac{\rho\,d\rho}{(\rho^2+H^2)^{3/2}}$$

$$= 2\pi\left[1 - H/(a^2+H^2)^{1/2}\right]$$

c) $H = a$, $\Phi = 2\pi\left[1 - \frac{a}{\sqrt{a^2+a^2}}\right] = 1.84$