2-3 a) Prove $\epsilon_{ijk}\,\epsilon_{klm} = \delta_{il}\cdot\delta_{jm} - \delta_{im}\,\delta_{jl}$

Consider $i = 1$.

$\epsilon_{1jk}\,\epsilon_{klm} = \emptyset, \quad j \neq 2, 3$

if $j = 2$, $k = 3$ and $\epsilon_{klm} = \emptyset$, $lm \neq 12, 21$

if $lm = 12$,

$\epsilon_{123}\,\epsilon_{312} = +1, \quad \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} = \delta\delta$

if $lm = 21$

$\epsilon_{123}\,\epsilon_{321} = -1, \quad \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} = -\delta_{11}\delta_{22} = -1$

Similarly if $j = 3$

b) Prove $(\bar{A}\times\bar{B})\cdot(\bar{C}\times\bar{D}) = (\bar{A}\cdot\bar{C})(\bar{B}\cdot\bar{D}) - (\bar{A}\cdot\bar{D})(\bar{B}\cdot\bar{C})$

$$(\bar{A}\times\bar{B})\cdot(\bar{C}\times\bar{D}) = \epsilon_{ijk}A_jB_k\,\epsilon_{ilm}\,C_lD_m$$
$$= (\epsilon_{ijk}\,\epsilon_{ilm})\,A_jB_kC_lD_m$$

use 2-27;

and $\epsilon_{ijk}\,\epsilon_{ilm} = \epsilon_{jki}\,\epsilon_{ilm}$;

$$(\epsilon_{ijk}\,\epsilon_{ilm})A_jB_kC_lD_m = (\epsilon_{jki}\,\epsilon_{ilm})A_jB_kC_lD_m$$
$$= (\delta_{jl}\cdot\delta_{km} - \delta_{jm}\delta_{kl})A_jB_kC_lD_m$$
$$= (\bar{A}\cdot\bar{C})(\bar{B}\cdot\bar{D}) - (\bar{A}\cdot\bar{D})(\bar{B}\cdot\bar{C})$$

where, e.g. $\delta_{jl}A_jC_l = \bar{A}\cdot\bar{C}$

2-3 a) Prove $\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

Consider $i=1$.

$\epsilon_{1jk}\epsilon_{klm} = 0$, $j \neq 2, 3$

if $j=2$, $k=3$ and $\epsilon_{3lm} = 0$, $lm \neq 12, 21$

if $lm = 12$,

$\epsilon_{123}\epsilon_{312} = +1$, $\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} = 1$

if $lm = 21$

$\epsilon_{123}\epsilon_{321} = -1$, $\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} = -\delta_{11}\delta_{22} = -1$

Similarly if $j = 3$

b) Prove $(\bar{A}\times\bar{B})\cdot(\bar{C}\times\bar{D}) = (\bar{A}\cdot\bar{C})(\bar{B}\cdot\bar{D}) - (\bar{A}\cdot\bar{D})(\bar{B}\cdot\bar{C})$

$(\bar{A}\times\bar{B})\cdot(\bar{C}\times\bar{D}) = \epsilon_{ijk}A_jB_k\epsilon_{ilm}C_lD_m$

$= (\epsilon_{ijk}\epsilon_{ilm})A_jB_kC_lD_m$

use 2-3a;

and $\epsilon_{ijk}\epsilon_{ilm} = \epsilon_{jki}\epsilon_{ilm}$,

$(\epsilon_{ijk}\epsilon_{ilm})A_jB_kC_lD_m = (\epsilon_{jki}\epsilon_{ilm})A_jB_kC_lD_m$

$= (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl})A_jB_kC_lD_m$

$= (\bar{A}\cdot\bar{C})(\bar{B}\cdot\bar{D}) - (\bar{A}\cdot\bar{D})(\bar{B}\cdot\bar{C})$

where, e.g. $\delta_{jl}A_jC_l = \bar{A}\cdot\bar{C}$