2-4 Prove $S_{ij} A_{ij} = 0$ if $S_{ij} = S_{ji}$ & $A_{ij} = -A_{ji}$

Let $Q \equiv S_{ij} A_{ij}$. [Also, $Q = S_{ji} A_{ij}$.]

$Q = S_{ji}(-A_{ji}) = -S_{ji}A_{ji}$ but indices are summed, i.e., $Q = -S_{ij}A_{ij} = -Q$

$\therefore Q = 0$.

2-5 a) 3×3 rotation matrix about z-axis:

$\bar{x} \to \hat{i}'x' + \hat{j}'y' + \hat{k}'z' = \hat{i}x + \hat{j}y + \hat{k}z$

$x' \to \hat{i}' \cdot \bar{x} = x\cos\theta + y\sin\theta$

$y' \to \hat{j}' \cdot \bar{x} = -x\sin\theta + y\cos\theta$

$z' = z$

then $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix}$; $R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) Verify R is orthogonal ($R^{-1} = R^T$)

$$R^T R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

i.e., $R^T R = \underline{\mathbb{1}}$ & $R^T = R^{-1}$

2-4 Prove $S_{ij}A_{ij} = 0$ if $S_{ij} = S_{ji}$ & $A_{ij} = -A_{ji}$

Let $\alpha = S_{ij}A_{ij}$. [Also, $\alpha = S_{ji}A_{ji}$.]

$\alpha = S_{ij}(-A_{ji}) = -S_{ji}A_{ji}$ but indices are summed, i.e., $\alpha = -S_{ij}A_{ij} = -\alpha$

$\therefore \alpha = 0$.

2-7 a) 3x3 rotation matrix about z-axis:

$\vec{x} \to \hat{i}'x' + \hat{j}'y' + \hat{k}'z' = \hat{i}x + \hat{j}y + \hat{k}z$

$x' \to \hat{i}' \cdot \vec{x} = x\cos\theta + y\sin\theta$

$y' \to \hat{j}' \cdot \vec{x} = -x\sin\theta + y\cos\theta$

$z' = z$

then $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix}$; $R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) Verify R is orthogonal ($R^{-1} = R^T$)

$R^T R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

i.e., $R^T R = \mathbb{1}$ & $R^T = R^{-1}$