

2-4 Prove $\sum_{i,j} S_{ij} A_{ij} = 0$ if $S_{ij} = S_{ji}$ & $A_{ij} = -A_{ji}$

Let $Q \equiv \sum_{i,j} S_{ij} A_{ij}$. [Also, $Q = \sum_{j,i} S_{ji} A_{ij}$]

$Q = \sum_{j,i} S_{ji} (-A_{ji}) = -\sum_{j,i} S_{ji} A_{ji}$ but indices are summed, i.e., $Q = -\sum_{i,j} S_{ij} A_{ij} = -Q$
 $\therefore Q = 0$.

2-5 a) 3×3 rotation matrix about z-axis:
 $\vec{x} \rightarrow \hat{i}'x' + \hat{j}'y' + \hat{k}'z' = \hat{i}x + \hat{j}y + \hat{k}z$
 $x' \rightarrow \hat{i}' \cdot \vec{x} = x \cos \theta + y \sin \theta$
 $y' \rightarrow \hat{j}' \cdot \vec{x} = -x \sin \theta + y \cos \theta$
 $z' = z$

then $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix}$; $R = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) Verify R is orthogonal ($R^{-1} = R^T$)

$$R^T R = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

i.e., $R^T R = \mathbb{1}$ & $R^T = R^{-1}$

5-1

Plane $z = z_0$ $\vec{x} = z_0 \vec{e}_3$ $\vec{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$\text{For } Q = z_0 \vec{e}_3 \text{ : } [\vec{A} Q = z_0 \vec{e}_3]$$

$$Q = z_0 \vec{e}_3 \text{ : } (\vec{A} Q) = z_0 \vec{e}_3 \text{ : } Q = z_0 \vec{e}_3 \text{ : } Q = z_0 \vec{e}_3 \text{ : } Q = z_0 \vec{e}_3$$

5-2

$$\vec{x} = \vec{e}_1 + \vec{e}_2 + \vec{e}_3 \text{ : } \vec{x} = \vec{e}_1 + \vec{e}_2 + \vec{e}_3 \text{ : } \vec{x} = \vec{e}_1 + \vec{e}_2 + \vec{e}_3$$

$$\vec{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ ; } \vec{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Verify $\vec{R}^{-1} = \vec{R}^T$

$$\vec{R}^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ ; } \vec{R}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vec{R}^T = \vec{R}^{-1}$$