2.6 a) $\bar{\nabla}\cdot(g\bar{F}) = \frac{\partial}{\partial x_i}(gF_i) = \frac{\partial g}{\partial x_i}F_i + g\frac{\partial F_i}{\partial x_i} = \bar{\nabla}g\cdot\bar{F} + g\bar{\nabla}\cdot\bar{F}$

b) $\bar{\nabla}\times(g\bar{F})$: $[\bar{\nabla}\times(g\bar{F})]_i = \epsilon_{ijk}\frac{\partial}{\partial x_j}(gF_k)$

$= \epsilon_{ijk}(\partial g/\partial x_j \; F_k + g\;\partial F_k/\partial x_j) = (\bar{\nabla}g\times\bar{F})_i + g(\bar{\nabla}\times\bar{F})_i$

c) $\bar{\nabla}\cdot(\bar{F}\times\bar{G}) = \frac{\partial}{\partial x_i}(\epsilon_{ijk}F_jG_k) = \epsilon_{ijk}\left[\frac{\partial F_j}{\partial x_i}G_k + F_j\frac{\partial G_k}{\partial x_i}\right]$

$= (\bar{\nabla}\times\bar{F})\cdot G - \bar{F}\cdot(\bar{\nabla}\times\bar{G})$

d) $[\bar{\nabla}\times(\bar{\nabla}\times\bar{F})]_i = \epsilon_{ijk}\frac{\partial}{\partial x_j}\epsilon_{klm}\,\partial F_m/\partial x_l$

$= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\frac{\partial^2 F_m}{\partial x_j\partial x_l}$

$= [\bar{\nabla}(\bar{\nabla}\cdot\bar{F}) - \nabla^2\bar{F}]_i$

2-7 $f(x,y) = Cxy$, $x \,\&\, y > 0$ but $xy \le a^2$

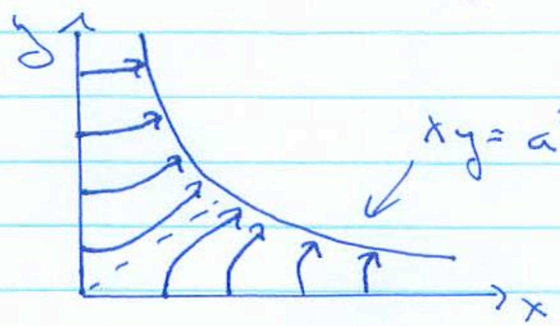
a) show $\nabla^2 f = 0$: $\nabla^2 f = \frac{\partial^2}{\partial x^2}(Cxy) + \frac{\partial^2}{\partial y^2}(Cxy) + \frac{\partial^2}{\partial z^2}(Cxy) =$

b) on x-axis, $f = 0$

on y-axis, $f = 0$

on $xy = a^2$, $f = Ca^2$

c) $\bar{\nabla}f = \hat{i}\,\partial f/\partial x + \hat{j}\,\partial f/\partial y + \hat{k}\,\partial f/\partial z = \hat{i}\,Cy + \hat{j}\,Cx$



Tangent curves must be $\perp$ to boundaries, which are equipotentials.

2.c) $\vec\nabla\cdot(\phi\vec F) = \frac{\partial}{\partial x_i}(\phi F_i) = \frac{\partial\phi}{\partial x_i}F_i + \phi\frac{\partial F_i}{\partial x_i} = \vec\nabla\phi\cdot\vec F + \phi\,\vec\nabla\cdot\vec F$

d) $\vec\nabla\times(\phi\vec F)$: $[\vec\nabla\times(\phi\vec F)]_i = \epsilon_{ijk}\frac{\partial}{\partial x_j}(\phi F_k)$

$= \epsilon_{ijk}\left(\frac{\partial\phi}{\partial x_j}F_k + \phi\frac{\partial F_k}{\partial x_j}\right) = (\vec\nabla\phi\times\vec F)_i + \phi(\vec\nabla\times\vec F)_i$

e) $\vec\nabla\cdot(\vec F\times\vec G) = \frac{\partial}{\partial x_i}(\epsilon_{ijk}F_jG_k) = \epsilon_{ijk}\left[\frac{\partial F_j}{\partial x_i}G_k + F_j\frac{\partial G_k}{\partial x_i}\right]$

$= (\vec\nabla\times\vec F)\cdot\vec G - \vec F\cdot(\vec\nabla\times\vec G)$

f) $[\vec\nabla\times(\vec\nabla\times\vec F)]_i = \epsilon_{ijk}\frac{\partial}{\partial x_j}\epsilon_{klm}\frac{\partial F_m}{\partial x_l}$

$= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\frac{\partial^2 F_m}{\partial x_j\partial x_l}$

$= [\vec\nabla(\vec\nabla\cdot\vec F) - \nabla^2\vec F]_i$

3.f $f(x,y) = Cxy$, $x,y > 0$ for $xy \le a^2$

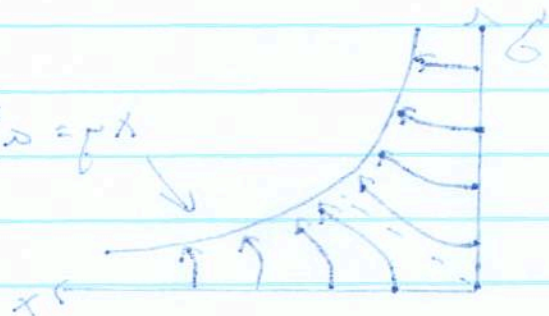
a) show $\nabla^2 f = 0$: $\nabla^2 f = \frac{\partial^2}{\partial x^2}(Cxy) + \frac{\partial^2}{\partial y^2}(Cxy) = 0$

b) on x-axis, $f = 0$

on y-axis, $f = 0$

on $xy = a^2$, $f = Ca^2$

c) $\vec\nabla f = \hat i\,\partial f/\partial x + \hat j\,\partial f/\partial y = \hat i\,Cy + \hat j\,Cx$



Tangent curves must be ⊥ to boundaries, which are equipotentials