

2.8 a) Find $\vec{\nabla}\phi$, $\phi(\vec{x}) = \frac{\vec{p} \cdot \vec{x}}{r^3}$, $\vec{p} = \text{constants}$.

write:

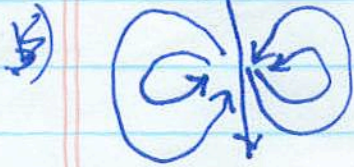
$$\phi = f g, \quad f = \vec{p} \cdot \vec{x} \quad \text{and} \quad g = 1/r^3$$

$$\vec{\nabla}\phi = f \vec{\nabla}g + g \vec{\nabla}f$$

$$\vec{\nabla}g = -3/r^4 \hat{r}$$

$$\vec{\nabla}f = \vec{p}$$

$$\vec{\nabla}\phi = \vec{p}/r^3 - \hat{r} \vec{p} \cdot \vec{x} / r^4 = \frac{(\vec{p} - \hat{r}(\hat{r} \cdot \vec{p}))}{r^3}$$



b) Find $\vec{\nabla} \times \vec{A}$, $\vec{A}(\vec{x}) = \vec{m} \times \vec{x} / r^3$, $\vec{m} = \text{const}$.

$$\vec{A} = \vec{F} g, \quad \vec{F} = \vec{m} \times \vec{x} \quad \text{and} \quad g = 1/r^3$$

$$\vec{\nabla} \times \vec{A} = \vec{\nabla} \times (\vec{F} g) = g \vec{\nabla} \times \vec{F} + \vec{\nabla} g \times \vec{F}$$

$$\vec{\nabla} \times \vec{F} = \vec{\nabla} \times (\vec{m} \times \vec{x}) = \underbrace{(\vec{x} \cdot \vec{\nabla}) \vec{m}}_{=0} - \underbrace{(\vec{m} \cdot \vec{\nabla}) \vec{x}}_{=-\vec{m}} + \underbrace{\vec{m}(\vec{\nabla} \cdot \vec{x})}_{+3\vec{m}} - \underbrace{\vec{x}(\vec{\nabla} \cdot \vec{m})}_{=0}$$

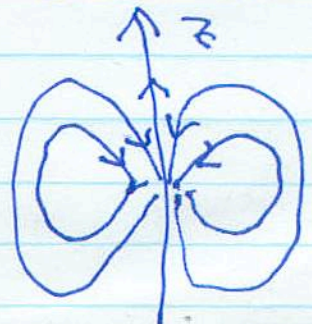
$$= 2\vec{m}$$

$$\vec{\nabla}g = -3\hat{r}/r^4; \quad \vec{\nabla}g \times \vec{F} = -3\hat{r}/r^4 \times (\vec{m} \times \vec{x}) = -3\hat{r}/r^4 (\vec{m}r - \vec{x}(\hat{r} \cdot \vec{m}))$$

$$= \frac{-3(\vec{m}r - \vec{x}(\hat{r} \cdot \vec{m}))}{r^4}$$

$$\vec{\nabla} \times \vec{A} = -\frac{1}{r^3} [\vec{m} - 3\hat{r}(\hat{r} \cdot \vec{m})]$$

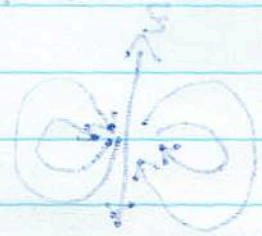
Curves are same but arrows opposite as part a:



5.2. Find $\nabla \phi = \vec{\nabla} \phi$ and $\nabla \cdot \vec{\nabla} \phi = \Delta \phi$ if $\phi = \frac{x \cdot \vec{q}}{r^2}$ and $\vec{q} = \text{const}$.

Let $\vec{q} = q \hat{z}$ and $r = \sqrt{x^2 + y^2 + z^2}$. Then $\phi = \frac{z}{r^2}$.
 $\nabla \phi = \frac{\partial \phi}{\partial x} \hat{x} + \frac{\partial \phi}{\partial y} \hat{y} + \frac{\partial \phi}{\partial z} \hat{z} = -\frac{2xz}{r^3} \hat{x} - \frac{2yz}{r^3} \hat{y} + \frac{r^2 - 2z^2}{r^3} \hat{z}$

$\Delta \phi = \nabla \cdot \nabla \phi = -\frac{2z}{r^3} - \frac{2z}{r^3} + \frac{r^2 - 2z^2}{r^3} = -\frac{2z}{r^3} + \frac{r^2 - 2z^2}{r^3} = \frac{r^2 - 4z^2}{r^3}$



5.3. Find $\nabla \times \vec{A}$ and $\nabla \cdot \vec{A}$ if $\vec{A} = \frac{\vec{r} \times \vec{m}}{r^2}$ and $\vec{m} = \text{const}$.

Let $\vec{m} = m \hat{z}$. Then $\vec{A} = \frac{1}{r^2} (\vec{r} \times \vec{m}) = \frac{1}{r^2} (-y \hat{x} + x \hat{y})$.
 $\nabla \times \vec{A} = \nabla \times \left(\frac{-y \hat{x} + x \hat{y}}{r^2} \right) = \frac{2\vec{m}}{r^3} = \frac{2m}{r^3} \hat{z}$
 $\nabla \cdot \vec{A} = \nabla \cdot \left(\frac{-y \hat{x} + x \hat{y}}{r^2} \right) = \frac{-1}{r^2} + \frac{1}{r^2} = 0$

5.4. Find $\nabla \times \vec{A}$ if $\vec{A} = \frac{\vec{r} \times \vec{r}}{r^2} = \frac{\vec{r} \times \vec{r}}{r^2} = 0$.

$\nabla \times \vec{A} = \nabla \times \left(\frac{\vec{r} \times \vec{r}}{r^2} \right) = 0$

