2.8 a) Find $\bar{\nabla}\phi$, $\quad \phi(\bar{x}) = \frac{\bar{p}\cdot\bar{x}}{r^3}$, $\quad \bar{p}$ = constant,

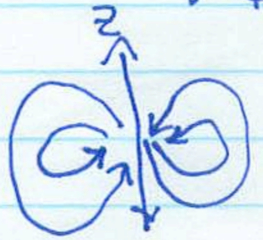
write:

$\phi = fg$, $\quad f = \bar{p}\cdot\bar{x}$ and $g = 1/r^3$

$\bar{\nabla}\phi = f\bar{\nabla}g + g\bar{\nabla}f$

$\bar{\nabla}g = -3/r^4\,\hat{r}$, $\qquad \bar{\nabla}f = \bar{p}$

$\bar{\nabla}\phi = \bar{p}/r^3 - \hat{r}\,\bar{p}\cdot\bar{x}/r^4 = \frac{(\bar{p} - \hat{r}(\hat{r}\cdot\bar{p}))}{r^3}$

b) 

b) Find $\bar{\nabla}\times\bar{A}$, $\quad \bar{A}(\bar{x}) = \bar{m}\times\bar{x}/r^3$, $\quad \bar{m}$ = const.

$\bar{A} = \bar{F}g$, $\quad \bar{F} = \bar{m}\times\bar{x}$ and $g = 1/r^3$

$\bar{\nabla}\times\bar{A} = \bar{\nabla}\times(\bar{F}g) = g\,\bar{\nabla}\times\bar{F} + \bar{\nabla}g\times\bar{F}$

$\bar{\nabla}\times\bar{F} = \bar{\nabla}\times(\bar{m}\times\bar{x}) = \underbrace{(\bar{x}\cdot\bar{\nabla})}_{=\emptyset}\bar{m} - \underbrace{(\bar{m}\cdot\bar{\nabla})\bar{x}}_{-\bar{m}} + \underbrace{\bar{m}(\bar{\nabla}\cdot\bar{x})}_{+3\bar{m}} - \bar{x}\underbrace{(\bar{\nabla}\cdot\bar{m})}_{=\emptyset}$

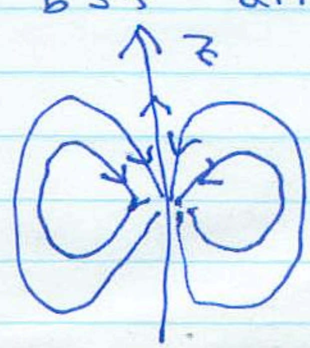
$= 2\bar{m}$

$\bar{\nabla}g = -3\hat{r}/r^4$; $\quad \bar{\nabla}g\times\bar{F} = -3\hat{r}/r^4\times(\bar{m}\times\bar{x}) = -3r^{-4}(mr - \bar{x}(\hat{r}$

$= \frac{-3(mr - \bar{x}(\hat{r}\cdot\bar{m}))}{r^4}$

$$\bar{\nabla}\times\bar{A} = -\frac{1}{r^3}\left[\bar{m} - 3\hat{r}(\hat{r}\cdot\bar{m})\right]$$

Curves are same but arrows opposite as part a:



2.8 a) Find $\nabla\phi$, $\phi(\vec{x}) = \frac{\vec{p}\cdot\vec{x}}{r^3}$, $\vec{p}$ = constant

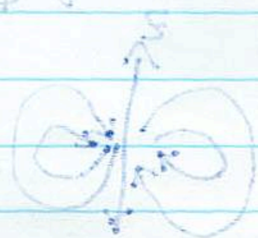
using:

$\phi = f g$, $f = \vec{p}\cdot\vec{x}$ and $g = 1/r^3$

$\vec{\nabla}\phi = f\vec{\nabla}g + g\vec{\nabla}f$

$\vec{\nabla}g = -3/r^4\,\hat{r}$ $\quad \vec{\nabla}f = \vec{p}$

$\vec{\nabla}\phi = \vec{p}/r^3 - 3\hat{r}\,\vec{p}\cdot\vec{x}/r^4 = \frac{(\vec{p} - \hat{r}(\hat{r}\cdot\vec{p}))}{r^3}$



b) Find $\vec{\nabla}\times\vec{A}$, $\vec{A}(\vec{x}) = \vec{m}\times\vec{x}/r^3$, $\vec{m}$ = const.

$\vec{A} = g\vec{F}$, $\vec{F} = \vec{m}\times\vec{x}$ and $g = 1/r^3$

$\vec{\nabla}\times\vec{A} = \vec{\nabla}\times(g\vec{F}) = \vec{\nabla}g\times\vec{F} + g\vec{\nabla}\times\vec{F}$

$\vec{\nabla}\times\vec{F} = \vec{\nabla}\times(\vec{m}\times\vec{x}) = \underbrace{\vec{m}(\vec{\nabla}\cdot\vec{x})}_{=3\vec{m}} - \underbrace{(\vec{m}\cdot\vec{\nabla})\vec{x}}_{=\vec{m}} + \underbrace{(\vec{x}\cdot\vec{\nabla})\vec{m}}_{=0} - \underbrace{\vec{x}(\vec{\nabla}\cdot\vec{m})}_{=0}$

$= 2\vec{m}$

$\vec{\nabla}g = -3\hat{r}/r^4$; $\vec{\nabla}g\times\vec{F} = -3\hat{r}/r^4\times(\vec{m}\times\vec{x}) = -3\frac{1}{r^4}(\vec{m}r - \vec{x}(\hat{r}\cdot\vec{m}))$

$= \frac{-3(\vec{m}r - \vec{x}(\hat{r}\cdot\vec{m}))}{r^4}$

$\vec{\nabla}\times\vec{A} = -\frac{1}{r^3}\left[\vec{m} - 3\hat{r}(\hat{r}\cdot\vec{m})\right]$

Curves are same but arrows opposite as part a: