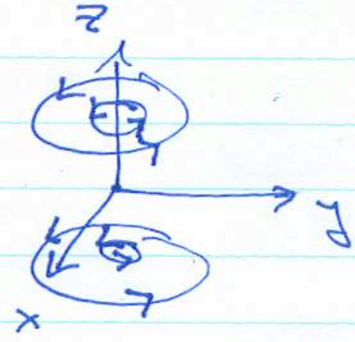
2-9 a) Sketch $\bar{F}(\bar{x}) = \hat{k} \times \vec{x}$:



$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ x & y & z \end{vmatrix} = -y\hat{i} + x\hat{j}$$

b) Find $\oint \bar{F} \cdot d\bar{l}$

In cylindrical coordinates, $x = a\cos\phi$

$y = a\sin\phi$

$\vec{F} = a\hat{\phi}$, $\hat{\phi} = -\hat{i}\sin\phi + \hat{j}\cos\phi$

$$\oint \bar{F} \cdot d\bar{l} = \oint a\hat{\phi} \cdot \hat{\phi}\, a\, d\phi = a^2 \int_0^{2\pi} d\phi = 2\pi a^2$$

where $d\bar{l} = a\,d\phi\,\hat{\phi}$

c) Stokes Thm: $\oint_c \bar{F} \cdot d\bar{l} = \int_s (\bar{\nabla} \times \bar{F}) \cdot d\bar{A}$

$$\bar{\nabla} \times \bar{F} = \bar{\nabla} \times (a\hat{\phi}) = \left(\tfrac{1}{r} \partial F_z / \partial\phi - \partial F_\phi / \partial z \right)\hat{r} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r} \right)\hat{\phi}$$
$$+ \left(\tfrac{1}{r} \frac{\partial}{\partial r}(r F_\phi) - \tfrac{1}{r} \partial F_r / \partial\phi \right)\hat{k}$$
$$= \frac{1}{r}\frac{\partial}{\partial r}(r^2)\,\hat{k} = \frac{2r}{r}\hat{k} = 2\hat{k}$$

$$\int_s (\bar{\nabla} \times \bar{F}) \cdot d\bar{A} = \int_s 2\hat{k} \cdot dA = 2A = 2\pi a^2$$

2-9 a) Sketch. $\vec{F}(\vec{x}) = \hat{k} \times \vec{x}$:

$$\hat{k} \times \vec{x} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ x & y & z \end{vmatrix} = -y\hat{x} + x\hat{y}$$

b) Find $\oint \vec{F} \cdot d\vec{l}$

In cylindrical coordinates, $x = a\cos\phi$, $y = a\sin\phi$

$$\vec{F} = a\,\hat{\phi}, \quad \hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

$$\oint \vec{F} \cdot d\vec{l} = \oint a\,\hat{\phi} \cdot \hat{\phi}\, a\, d\phi = a^2 \int_0^{2\pi} d\phi = 2\pi a^2$$

where $d\vec{l} = a\,d\phi\,\hat{\phi}$

c) Stokes Thm: $\oint_C \vec{F} \cdot d\vec{l} = \int_S (\nabla \times \vec{F}) \cdot d\vec{A}$

$$\nabla \times \vec{F} = \nabla \times (a\hat{\phi}) = \left(\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{\phi}$$
$$+ \left(\frac{1}{r}\frac{\partial}{\partial r}(r F_\phi) - \frac{1}{r}\frac{\partial F_r}{\partial \phi}\right)\hat{k}$$
$$= \frac{1}{r}\frac{\partial}{\partial r}(r^2)\,\hat{k} = \frac{2r}{r}\,\hat{k} = 2\hat{k}$$

$$\int_S (\nabla \times \vec{F}) \cdot d\vec{A} = \int_S 2\hat{k} \cdot d\vec{A} = 2A = 2\pi a^2$$