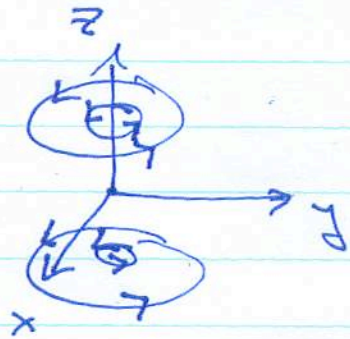


2-9 a) Sketch $\vec{F}(\vec{x}) = \hat{k} \times \vec{x}$:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ x & y & z \end{vmatrix} = -y\hat{i} + x\hat{j}$$



b) Find $\oint \vec{F} \cdot d\vec{l}$

In cylindrical coordinates, $x = a \cos \phi$

$$y = a \sin \phi$$

$$\vec{F} = a \hat{\phi}, \quad \hat{\phi} = -\hat{i} \sin \phi + \hat{j} \cos \phi$$

$$\oint \vec{F} \cdot d\vec{l} = \oint a \hat{\phi} \cdot \hat{\phi} a d\phi = a^2 \int_0^{2\pi} d\phi = 2\pi a^2$$

where $d\vec{l} = a d\phi \hat{\phi}$

c) Stokes Thm: $\oint_C \vec{F} \cdot d\vec{l} = \int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{A}$

$$\vec{\nabla} \times \vec{F} = \vec{\nabla} \times (a \hat{\phi}) = \left(\frac{1}{r} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) \hat{r} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r} \right) \hat{\phi}$$

$$+ \left(\frac{1}{r} \frac{\partial}{\partial r} (r F_\phi) - \frac{1}{r} \frac{\partial F_r}{\partial \phi} \right) \hat{k}$$

$$= \frac{1}{r} \frac{\partial}{\partial r} (r^2) \hat{k} = \frac{2r}{r} \hat{k} = 2 \hat{k}$$

$$\int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{A} = \int_S 2 \hat{k} \cdot d\vec{A} = 2A = 2\pi a^2$$

S-2

a) $\vec{r} \times \vec{K} = (\vec{r} \cdot \vec{K}) \vec{r} - r^2 \vec{K}$

$$\vec{r} \times \vec{K} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ 5 & 2 & 3 \end{vmatrix}$$



b) Find $\vec{r} \cdot \vec{K}$

In cylindrical coordinates (r, θ, z) $\vec{r} = r\hat{\rho} + z\hat{z}$

$\vec{K} = 5\hat{x} + 2\hat{y} + 3\hat{z}$

$$\vec{r} \cdot \vec{K} = (r\hat{\rho} + z\hat{z}) \cdot (5\hat{x} + 2\hat{y} + 3\hat{z})$$

$$= r(5\cos\theta + 2\sin\theta) + 3z$$

c) $\vec{A} \cdot (\vec{r} \times \vec{K}) = \vec{A} \cdot \vec{r} \cdot \vec{K} - r^2 \vec{A} \cdot \vec{K}$

$$\vec{A} \cdot (\vec{r} \times \vec{K}) = \begin{vmatrix} 1 & 0 & 0 \\ 5 & 2 & 3 \\ 5 & 2 & 3 \end{vmatrix} = 1(6 - 9) - 0(15 - 15) + 0(10 - 10) = -3$$

$$\vec{A} \cdot (\vec{r} \times \vec{K}) = \vec{A} \cdot \vec{r} \cdot \vec{K} - r^2 \vec{A} \cdot \vec{K}$$