3-20 a) $V(\vec{x}) = \vec{C}\cdot\vec{x} = C_x x + C_y y + C_z z$

$\vec{E} = -\vec{\nabla}V = -(C_x + C_y + C_z) = -\vec{C}$

b) if $\vec{E} = E_0 \hat{k}$, $\vec{C} = -E_0 \hat{k}$ + constant

$V = -E_0 z$

3-21

Potential $V(x,y,z) = \int \frac{dq}{4\pi\varepsilon_0 r'}$

$dq = \lambda\, d\phi\, \rho$

$= \left(\frac{Q}{2\pi a}\right)(\rho\, d\phi)$

$V = \frac{Q/2\pi a}{4\pi\varepsilon_0}\int \frac{\rho\, d\phi}{\sqrt{z^2+\rho^2}} = \frac{\frac{Q}{2\pi a}\,\rho\cdot 2\pi}{4\pi\varepsilon_0\sqrt{z^2+\rho^2}}$

but $\rho = a$

$V = \frac{\frac{Q}{2\pi a}\cdot 2\pi a}{4\pi\varepsilon_0\sqrt{z^2+a^2}} = \frac{Q}{4\pi\varepsilon_0\sqrt{z^2+a^2}}$

$\vec{E} = -\vec{\nabla}V = \frac{-Q}{4\pi\varepsilon_0}\left(z^2+a^2\right)^{-3/2}\cdot\frac{-1}{2}\cdot 2z\,\hat{k}$

$= \frac{Qz}{4\pi\varepsilon_0\left(z^2+a^2\right)^{3/2}}\,\hat{k}$

3-22