3-24 $\vec{E} = \hat{i}(2x^2 - 2xy - 2y^2) + \hat{j}(-x^2 - 4xy + y^2)$

$$\bar{\nabla} \times \bar{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & \emptyset \end{vmatrix} = \hat{i}(\emptyset - \emptyset) + \hat{j}(\emptyset - \emptyset) + \hat{k}(-2x - 4y - (-2x - 4y)) = \emptyset$$

e.g. $\bar{E}$ is irrotational.

$\therefore \ \bar{E} = -\bar{\nabla} V$

$-\partial V/\partial x = 2x^2 - 2xy - 2y^2 \rightarrow V = -\frac{2}{3}x^3 + x^2 y + 2xy^2 + c_1$

$-\partial V/\partial y = -x^2 - 4xy + y^2 \rightarrow V = -\frac{1}{3}y^3 + 2xy^2 + x^2 y + c_2$

$V = -2/3\, x^3 + x^2 y + 2xy^2 - 1/3\, y^3 + C_0$

$\bar{\nabla} \cdot \bar{E} = (4x - 2y) + (-4x + 2y) = \emptyset$

3-25 $V(r, \theta) = \dfrac{c_1 \cos^2\theta + c_2}{r^3}$

$\bar{\nabla} \cdot \bar{E} = \rho/\epsilon_0 = \emptyset$ for charge-free region

$-\nabla^2 V = \emptyset$

$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \partial V/\partial r\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right)$ for $V \neq V(\phi)$

$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(-3rV\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \cdot \frac{2c_1 \cos\theta}{r^3}\right)$

$= 6(C_1 \cos^2\theta + c_2)/r^5 - \frac{2C_1}{r^5}(3\cos^2\theta - 1) = \emptyset$

or

$C_1 = -3C_2$