The potential at the origin, V(0) is given by:

$$V(0) = \int \frac{\lambda\, ds}{4\pi\epsilon_0\, r}$$

Where the arclength ds = $\sqrt{(dr)^2 + r^2 (d\theta)^2}$. Since r=a(1 - θ/4π), dr = -a/4π dθ (or turning it around, dθ= -4π/a dr). Then ds= dr $\sqrt{1 + 16\pi^2 r^2 / a^2}$. This in turn gave

$$V(0) = \frac{\lambda}{4\pi\epsilon_0} \int_{a/2}^{a} \frac{dr}{r} \sqrt{1 + 16\pi^2 r^2 / a^2} .$$

Subsituting u= r/a (and therefore, du= dr/a), and writing dr/r = (dr/a)/(r/a) = du/u,

$$V(0) = \frac{\lambda}{4\pi\epsilon_0} \int_{1/2}^{1} \sqrt{1 + 16\pi^2 u^2}\, \frac{du}{u}$$

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Integrate[Sqrt[1 + 16 π^2 u^2]/u, {u, 0.5, 1}]
6.32283
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$$V(0) = 6.32283 \frac{\lambda}{4\pi\epsilon_0}$$