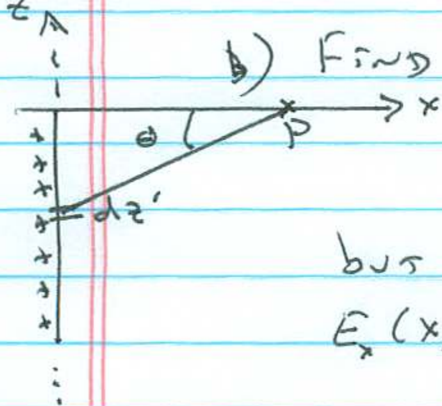
3-8 A semi-infinite wire on the negative z-axis has λ, $0 > z > -\infty$.

a) Find $E(0, 0, z)$ for $z > 0$

$$dE_z = \frac{\lambda\, dz'}{4\pi\varepsilon_0 (z - z')^2} \quad ; \quad E_z = \int_{-\infty}^{0} \frac{\lambda\, dz'}{4\pi\varepsilon_0 (z - z')^2} = \frac{\lambda}{4\pi\varepsilon_0 z}$$

$$\vec{E}_z = \frac{\lambda}{4\pi\varepsilon_0 z}\,\hat{z}$$ by symmetry.

b) Find $E(x, 0, 0)$



$$\vec{E} = E_x\,\hat{i} + E_y\,\hat{j} + E_z\,\hat{k}$$

but $E_y = 0$ by symmetry.

$$E_x(x, 0, 0) = \frac{1}{4\pi\varepsilon_0} \int_{-\infty}^{0} \frac{\lambda\, dz'}{(x^2 + z'^2)} \cos\theta$$

but $\cos\theta = \frac{x}{\sqrt{x^2 + z'^2}}$ $\rightarrow$ $E_x = \frac{\lambda x}{4\pi\varepsilon_0} \int_{-\infty}^{0} \frac{dz'}{(x^2 + z'^2)^{3/2}}$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 x}$$

$$E_z = \frac{1}{4\pi\varepsilon_0} \int_{-\infty}^{0} \frac{\lambda\, dz'}{(x^2 + z'^2)} \sin\theta$$ but $\sin\theta = \frac{-z'}{\sqrt{x^2 + z'^2}}$

$$E_z(x, 0, 0) = \frac{-\lambda}{4\pi\varepsilon_0} \int_{-\infty}^{0} \frac{z'\, dz'}{(x^2 + z'^2)^{3/2}} = \frac{\lambda}{4\pi\varepsilon_0 x}$$

e.g. $|\vec{E}_x| = |\vec{E}_z|$