

4.26 Consider a charge q halfway between two grounded conducting plates. The electric field can be calculated by the image method, with an infinite number of images. On the midplane the field points away from q , by symmetry. Using Mathematica plot the contribution to the electric field from the images charges (i.e., the full field minus the Coulomb field of the real charge) on the midplane, as a function of the distance from q . At what point on the midplane is the field due to the conducting planes maximum in magnitude?

To do this, setup a coordinate system with q at the origin and two conducting plates parallel to the yz plane at $x = \pm a$. There are an infinite number of image charges, which we can label q_n where $n = \pm 1, \pm 2, \pm 3, \dots$ and $q_n = (-1)^n q$ is located at $x_n = 2na$.

The midplane is the yz plane, $x=0$. There, at a distance r from the origin, the electric field is

$$E_r = \frac{1}{4\pi\epsilon_0} \left\{ \frac{q}{r^2} - \frac{2qr}{[r^2 + (2a)^2]^{3/2}} + \frac{2qr}{[r^2 + (4a)^2]^{3/2}} - \dots \right\}$$

$$= \frac{q}{4\pi\epsilon_0 r^2} + \frac{2qr}{4\pi\epsilon_0} \sum_{n=1}^{\infty} \frac{(-1)^n}{[r^2 + (2na)^2]^{3/2}}$$

where the second term is the field due to the surface charge of the conducting plates. The figure below shows a graph of this field:

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In[15]:= Plot[r Sum_{k=1}^{10} 2 * ((-1)^k) / (r^2 + 4 k^2)^(3/2), {r, 0, 10}]
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