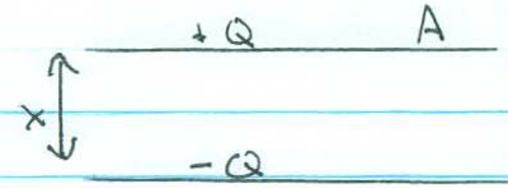
4-3 $\quad C = \varepsilon_0 A/x$



a) $F/A = \sigma E_1 = -\sigma^2/2\varepsilon_0$

$= -Q^2/2\varepsilon_0 A^2$ defining up as +.

Then $W = (-F)\Delta x$

$= Q^2 \Delta x / 2\varepsilon_0 A$

$\Delta U = W = Q^2 \Delta x / 2\varepsilon_0 A$

writing $V_0 = Q/c$,

$$\Delta U = (V_0 \varepsilon_0 A / x)^2 \cdot \frac{\Delta x}{2\varepsilon_0 A} = \frac{\varepsilon_0 V_0^2 A}{2x^2} \Delta X$$

or using $\Delta U = Q^2/2c_f - Q^2/2c_i$

$$= \frac{Q^2}{2\varepsilon_0 A} \Delta X$$

b) If $V = V_0$ is kept constant

$$W_{ext} + W_{batt} = \Delta U$$

but $\Delta U = \frac{1}{2} C_f V_0^2 - \frac{1}{2} C_i V_0^2$

$$= \frac{\varepsilon_0 A V_0^2}{2x^2} \left(\frac{1}{x+\Delta x} - \frac{1}{x} \right)$$

expanding to lowest order

$$\Delta U \approx -\frac{\varepsilon_0 A V_0^2}{2x^2} \Delta x$$ or decrease in energy

Note $W_{batt} = (Q_f - Q_i) V_0$

$$= (C_f - C_i) V_0^2 = 2\Delta U$$

$$= -\frac{\varepsilon_0 A V_0^2}{x^2} \Delta X$$

$$W_{ext} = F_{ext} \Delta X$$

$$= \frac{\varepsilon_0 A V_0^2}{2x^2} \Delta X$$