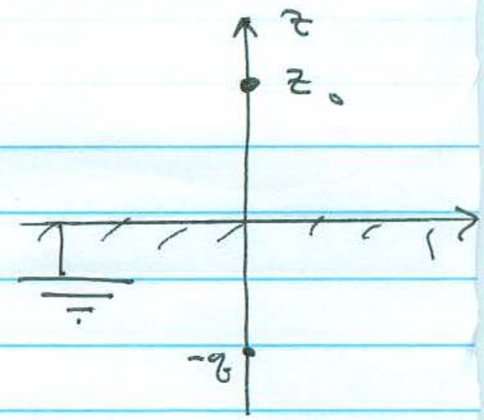
4-6 Charge q is at $(\emptyset, \emptyset, z_0)$ above grounded conductor.

a) Find induced charge.



The $\vec{E}$ has same form as for a point charge $-q$ at $(\emptyset, \emptyset, -z_0)$.

Surface charge has $\sigma(r) = \epsilon_0 E_z(x, y)$

where $r \equiv \sqrt{x^2 + y^2}$

$$\sigma(r) = \frac{-2q}{4\pi(r^2 + z_0^2)} \cos\theta = \frac{-2q\, z_0}{4\pi(r^2 + z_0^2)^{3/2}}$$

$$Q = \int_0^\infty \sigma(r)\, dA = \int_0^\infty \sigma(r) \cdot 2\pi r\, dr$$

$$= -q z_0 \int_0^\infty \frac{r\, dr}{(r^2 + z_0^2)^{3/2}}$$

We have done this integral a few times.

Let $u = r^2 + z_0^2$, $du = 2r\,dr$

$$Q = \frac{-q z_0}{2} \int_{z_0^2}^\infty du/u^{3/2} = \frac{-q z_0}{2} \cdot 2\left(\frac{-1}{\infty} + \frac{1}{z_0}\right)$$

$= -q$

b) find R such that

$$\int_0^R \sigma(r)\, 2\pi r\, dr = -q/2$$

$$-q/2 = q z_0 (r^2 + z_0^2)^{-1/2} \Big|_0^R$$

$$= q z_0 \left[\frac{1}{\sqrt{R^2 + z_0^2}} - \frac{1}{z_0}\right]$$

$$1/2 = 1 - \frac{z_0}{\sqrt{R^2 + z_0^2}} \rightarrow \frac{z_0}{\sqrt{R^2 + z_0^2}} = 1/2$$

$$z_0^2 / R^2 + z_0^2 = \frac{1}{4} \;;\; z_0^2 = \frac{R^2}{4} + \frac{z_0^2}{4}$$

$$\frac{3}{4} z_0^2 = R^2/4 \Longrightarrow R = \sqrt{3}\, z_0$$