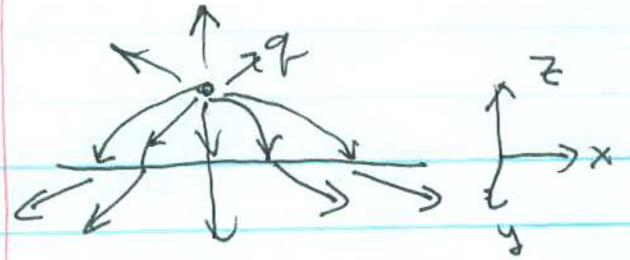
6-18



q is above dielectric plane. Show $q + q' = q''$ & $q - q' = Kq''$

eqn. 6.67 gives $$V_{above} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{q'}{\sqrt{x^2+y^2+(z+d)^2}}\right]$$

eqn 6.68 gives $$V_{below} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q''}{\sqrt{x^2+y^2+(z-d)^2}}\right]$$

$\vec{E}_t$ is continuous

$$\therefore \quad \frac{-\partial V_{above}}{\partial x} = \frac{-\partial V_{below}}{\partial x} \quad \text{at } z = 0$$

$$\frac{qx + q'x}{[x^2+y^2+d^2]^{3/2}} = \frac{q''x}{[x^2+y^2+d^2]^{3/2}} \Rightarrow q + q' = q''$$

$\vec{D}\cdot\hat{n}$ is continuous at $z = 0$:

$$\vec{D} = \varepsilon\vec{E} \longrightarrow -\varepsilon_0 \partial V_{above}/\partial z = -\varepsilon \frac{\partial V_{below}}{\partial z}$$

$$\frac{q(-d) + q'(d)}{(x^2+y^2+d^2)^{3/2}} = \frac{\varepsilon}{\varepsilon_0}\frac{q''(-d)}{(x^2+y^2+d^2)^{3/2}}$$

$$q - q' = Kq''$$