

6-19 q is a height z above a planar dielectric

a) $\vec{F} = q \vec{E}_{q'}$

$\vec{E} = \vec{E}_q + \vec{E}_{q'}$ but charges do not exhibit self forces

$$\vec{E}_{q'} = \frac{q'}{4\pi\epsilon_0 r^2} ; \quad q' = \frac{k-1}{k+1} q \quad (\text{eqn. 6.64})$$

$$r^2 = (2z)^2 = 4z^2$$

$$\vec{F} = \frac{-q^2 (k-1)}{4\pi\epsilon_0 (k+1) z^2}$$

b) Surface charge density:

$$\vec{E} \cdot \hat{n} = \frac{\sigma}{\epsilon_0}$$

$$E_n^{\text{above}} - E_n^{\text{below}} = \frac{\sigma}{\epsilon_0} \quad \hat{n} = \hat{k}$$

$$\sigma = \epsilon_0 \left[-\frac{\partial V^{\text{above}}}{\partial z} + \frac{\partial V^{\text{below}}}{\partial z} \right]_{z=0}$$

$$= \frac{\epsilon_0}{4\pi\epsilon_0} \left[\frac{-q d + q' d}{(x^2 + y^2 + d^2)^{3/2}} - \frac{q'' d}{(x^2 + y^2 + d^2)^{3/2}} \right]$$

$$= \frac{-2dq}{4\pi(x^2 + y^2 + z^2)^{3/2}} \left(\frac{k-1}{k+1} \right)$$