6-4 Take q at $(0,0,0)$ & $\bar{p} = p_0(\hat{\imath}\sin\phi + \hat{k}\cos\phi)$ at $(0,0,z_0)$.

a) $U(\bar{x}) = -\bar{p}\cdot\bar{E}(\bar{x})$

$$\vec{F}_{dipole} = -\bar{\nabla}U = \bar{\nabla}(\bar{p}\cdot\bar{E}(\bar{x}))$$

Alternately,

$$\bar{E} = -\bar{\nabla}V$$

$$\bar{\nabla}(\bar{p}\cdot\bar{E}) = -\hat{e}_i \frac{\partial}{\partial x_i} p_j \frac{\partial V}{\partial x_j} = -\hat{e}_i p_j \frac{\partial^2 V}{\partial x_i x_j}$$

$$= \left(p_j \frac{\partial}{\partial x_j}\right)\left(-\hat{e}_i \partial V/\partial x_i\right) = (\bar{p}\cdot\bar{\nabla})\bar{E}$$

b) $\bar{E} = q\bar{x}/4\pi\varepsilon_0 r^3$

$$\bar{p}\cdot\bar{E} = \frac{p_0 q}{4\pi\varepsilon_0 r^3}\left[x\sin\phi + z\cos\phi\right]$$

$$\bar{F} = \bar{\nabla}(\bar{p}\cdot\bar{E}) = \frac{p_0 q}{4\pi\varepsilon_0}\left[\frac{-3\hat{r}}{r^4}(x\sin\phi + z\cos\phi) + \frac{1}{r^3}(\hat{\imath}\sin\phi + \hat{k}\cos\phi)\right]$$

but if $\bar{p}$ has position $(0,0,z_0)$, $\hat{r} = \hat{k}$, $r = z_0$

$$\bar{F} = \frac{p_0 q}{4\pi\varepsilon_0 z_0^3}\left[-2\hat{k}\cos\phi + \hat{\imath}\sin\phi\right]$$

$$\bar{T} = \bar{p}\times\bar{E} = p_0\left[\hat{\imath}\sin\phi + \hat{k}\cos\phi\right]\times\frac{q\bar{x}}{4\pi\varepsilon_0 r^3}$$

$$= -\frac{p_0 q}{4\pi\varepsilon_0 z_0^3}\sin\phi\,\hat{\jmath}$$

$$\bar{E}_{dip} = \frac{1}{4\pi\varepsilon_0 r^3}\left[3\hat{r}(\hat{r}\cdot\bar{p}) - \bar{p}\right]$$

$$\bar{F} = q\bar{E} = \frac{q p_0}{4\pi\varepsilon_0 z^3}\left[2\hat{k}\cos\phi - \hat{\imath}\sin\phi\right]$$